

DIBOSON PRODUCTION AT NNLO

Andreas v. Manteuffel



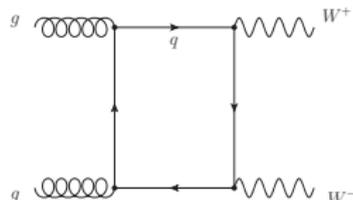
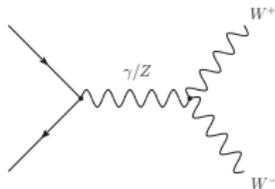
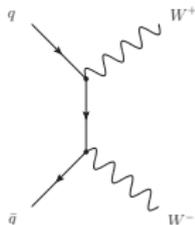
Seminar Fermilab
25. June 2015

VECTOR BOSON PAIR PRODUCTION AT LHC

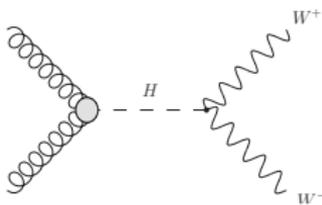
$pp \rightarrow VV' + X \rightarrow 4 \text{ leptons} + X$, where $VV' = ZZ, W^+W^-, \gamma^*\gamma^*, ZW^\pm, Z\gamma^*, W^\pm\gamma^*$

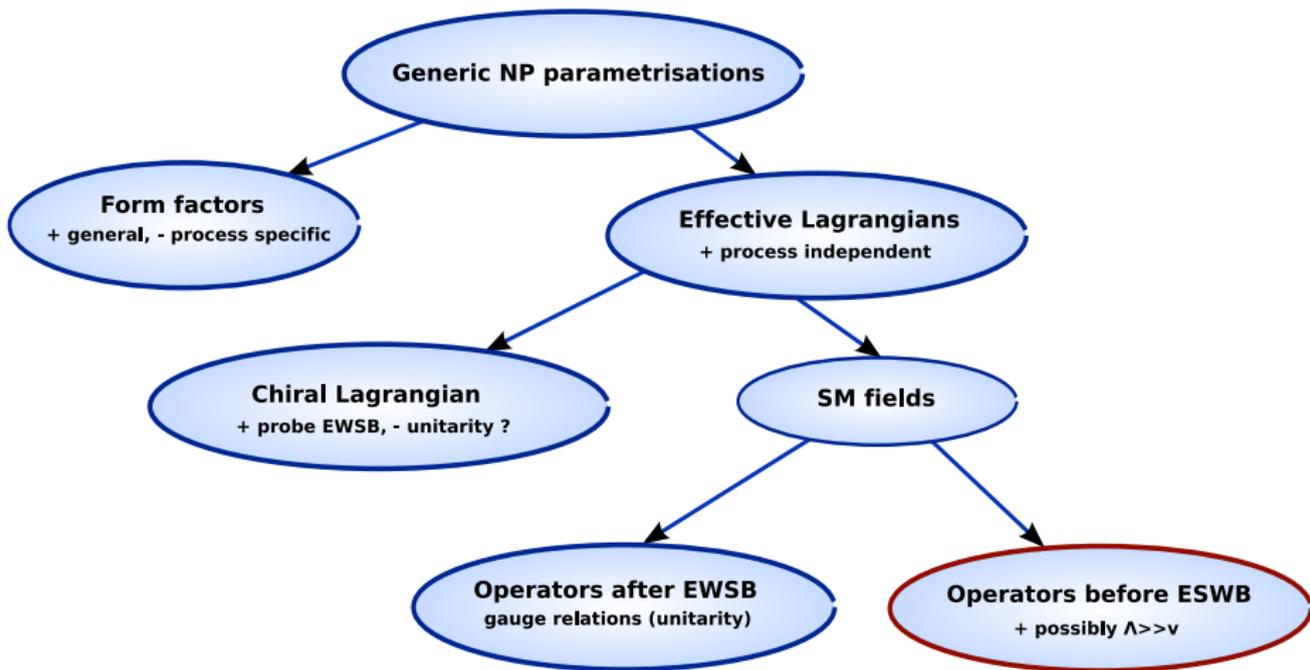
- sensitive to details of EWSB
- possible NP contributions at tree or loop level

e.g. W^+W^- production:



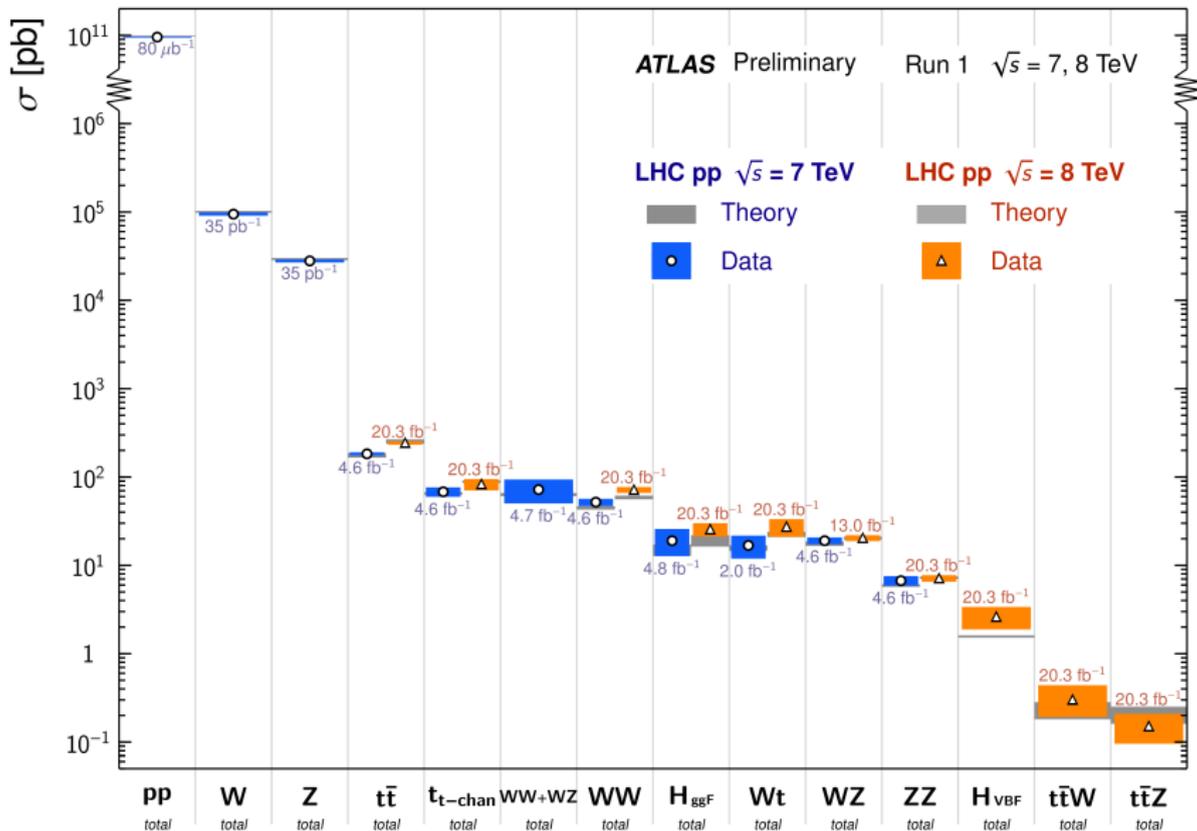
important background to Higgs signals:

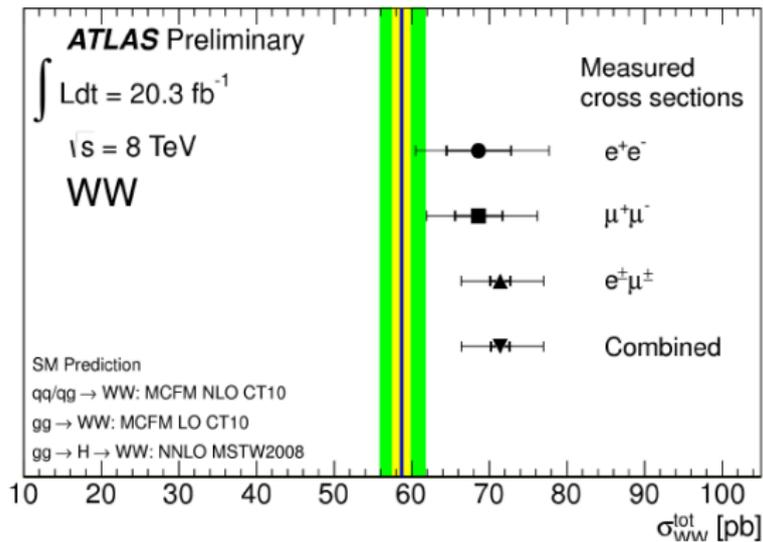


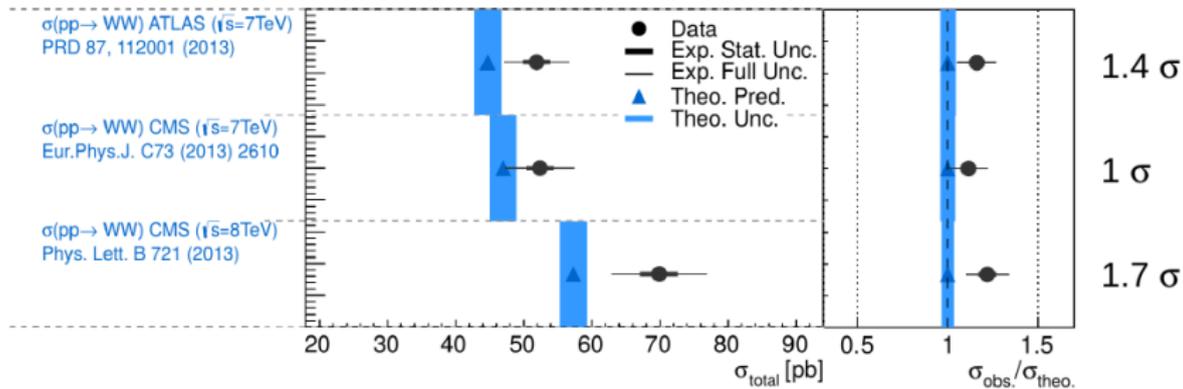


Standard Model Total Production Cross Section Measurements

Status: July 2014







new physics ?

make sure to understand SM prediction !

QCD approx. NNLO and electroweak NLO:

- gg initiated (one-loop only): [Binoth et al. (2005,2008); Duhrssen et al. (2005); Amettler et al. (1985); van der Bij, Glover (1988); Adamson, de Florian, Signer (2000)]
- high energy WW : [Chachamis, Czakon, Eiras (2008)]
- electroweak NLO: [Hollik, Meier (2004); Accomando, Denner, Meier (2005); Bierweiler, Kasprzik, Kühn, Uccirati (2012); Baglio, Ninh, Weber (2013); Billoni, Dittmaier, Jäger, Speckner (2013)]

QCD full NNLO:

- equal mass integrals: [Gehrmann, Tancredi, Weihs '13; Gehrmann, AvM, Tancredi, Weihs '14]
- non-equal mass integrals: [Henn, Melnikov, Smirnov '14; Caola, Henn, Melnikov, Smirnov '14]; [Papadopoulos, Tammasini, Wever '14]; [Gehrmann, AvM, Tancredi '15]
- amplitudes $q\bar{q}' \rightarrow VV'$: [Caola, Henn, Melnikov, Smirnov '14]; [Gehrmann, AvM, Tancredi '15]
- amplitudes $gg \rightarrow VV'$: [Caola, Henn, Melnikov, Smirnov '15]; [AvM, Tancredi '15]
- $ZZ@NNLO$ [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi, Weihs '14]
- $WW@NNLO$ [Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14]
- partial results: [Anastasiou, Cancino, Chavez, Duhr, Lazopoulos, Mistlberger, Müller '14]

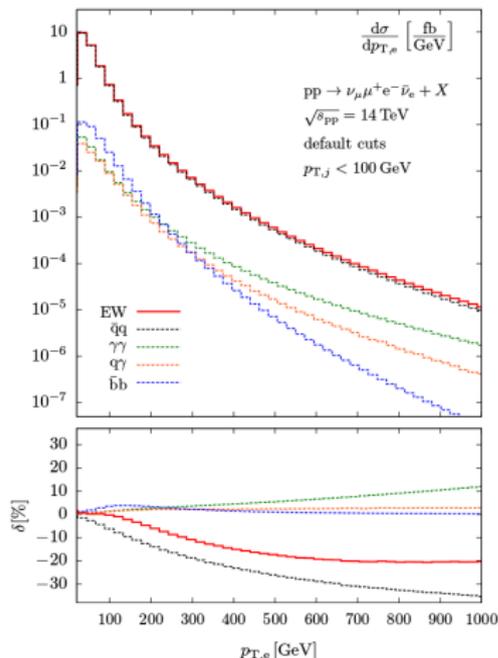
ELECTROWEAK NLO EFFECTS

full EW NLO to $4l$ in double pole approx. by [Billoni, Dittmaier, Jäger, Speckner (2013)]:

- small for total cross section

	$\sigma_{\bar{q}q}^{\text{LO}}$ [fb]	$\delta_{\bar{q}q}$ [%]	$\delta_{q\gamma}$ [%]	$\delta_{\gamma\gamma}$ [%]	$\delta_{\bar{b}b}$ [%]
LHC14	412.5(1)	-2.70(2)	0.566(5)	0.7215(4)	1.685(1)
LHC8	236.83(5)	-2.76(1)	0.470(3)	0.8473(3)	0.8943(3)
ATLAS cuts	163.84(4)	-2.96(1)	-0.264(5)	1.0221(5)	0.9519(4)

- significant for distributions



ingredients for $VV' + X$ production at NNLO QCD:

	LO	NLO	NNLO	\mathcal{M}_0	\mathcal{M}_1	\mathcal{M}_2
$2 \rightarrow 2$	$\mathcal{M}_0^* \mathcal{M}_0$ $q\bar{q}$	$\mathcal{M}_0^* \mathcal{M}_1$ $q\bar{q}$	$\mathcal{M}_0^* \mathcal{M}_2, \mathcal{M}_1^* \mathcal{M}_1$ $q\bar{q}, g\bar{g}$			
$2 \rightarrow 3$	-	$\mathcal{M}_0^* \mathcal{M}_0$ $q\bar{q}, qg$	$\mathcal{M}_0^* \mathcal{M}_1$ $q\bar{q}, qg$			
$2 \rightarrow 4$	-	-	$\mathcal{M}_0^* \mathcal{M}_0$ $q\bar{q}, qg, g\bar{g}$			

note: some channels contribute only at higher orders:

- qg starting at NLO
- $g\bar{g}$ starting at NNLO → control error by computing N³LO contributions from this channel

subtraction terms: up to 2 unresolved partons needed

- **q_T subtraction:** [Catani, Grazzini '07; Catani, Cieri, de Florian, Ferrera, Grazzini '13]
- N -jettiness subtraction: [Boughezal, Foecke, Liu, Petriello '15; Boughezal, Foecke, Giele, Liu, Petriello '15; Gaunt, Stahlhofen, Tackmann, Walsh '15]
- antenna subtraction: [Gehrmann-De Ridder, Gehrmann, Glover '05]
- sector-improved subtraction: [Czakon '10]

Expertise for all ingredients crucial for VV @ NNLO QCD



STEFAN KALLWEIT



ERICH WEIHS



LORENZO TANCREDI



ANDREAS VON MANTEUFFEL



THOMAS GEHRMANN



FABIO CASCIOLI



STEFANO POZZORINI



PHILIPP MAZERHÖFER



ALESSANDRO TORRE



MASSIMILIANO GRAZZINI



DIRK RATHLEY

LORENTZ STRUCTURES FOR VV' AMPLITUDE

VV' amplitude:

$$S^{\mu\nu}(p_1, p_2, p_3) = \sum_j A_j(s, t, p_3^2, p_4^2) T_j^{\mu\nu}$$

$q\bar{q}'$ channel:

$$\begin{aligned} T_1^{\mu\nu} &= \bar{u}(p_2) \not{p}_3 u(p_1) p_1^\mu p_1^\nu, & T_2^{\mu\nu} &= \bar{u}(p_2) \not{p}_3 u(p_1) p_1^\mu p_2^\nu, \\ T_3^{\mu\nu} &= \bar{u}(p_2) \not{p}_3 u(p_1) p_2^\mu p_1^\nu, & T_4^{\mu\nu} &= \bar{u}(p_2) \not{p}_3 u(p_1) p_2^\mu p_2^\nu, \\ T_5^{\mu\nu} &= \bar{u}(p_2) \gamma^\mu u(p_1) p_1^\nu, & T_6^{\mu\nu} &= \bar{u}(p_2) \gamma^\mu u(p_1) p_2^\nu, \\ T_7^{\mu\nu} &= \bar{u}(p_2) \gamma^\nu u(p_1) p_1^\mu, & T_8^{\mu\nu} &= \bar{u}(p_2) \gamma^\nu u(p_1) p_2^\mu, \\ T_9^{\mu\nu} &= \bar{u}(p_2) \gamma^\mu \not{p}_3 \gamma^\nu u(p_1), & T_{10}^{\mu\nu} &= \bar{u}(p_2) \gamma^\nu \not{p}_3 \gamma^\mu u(p_1). \end{aligned}$$

gg channel:

$$\begin{aligned} T_1^{\mu\nu} &= \epsilon_1 \cdot \epsilon_2 g^{\mu\nu}, & T_2^{\mu\nu} &= \epsilon_1^\mu \epsilon_2^\nu, & T_3^{\mu\nu} &= \epsilon_1^\nu \epsilon_2^\mu, & T_4^{\mu\nu} &= \epsilon_1 \cdot \epsilon_2 p_1^\mu p_1^\nu, \\ T_5^{\mu\nu} &= \epsilon_1 \cdot \epsilon_2 p_1^\mu p_2^\nu, & T_6^{\mu\nu} &= \epsilon_1 \cdot \epsilon_2 p_2^\mu p_1^\nu, & T_7^{\mu\nu} &= \epsilon_1 \cdot \epsilon_2 p_2^\mu p_2^\nu, & T_8^{\mu\nu} &= \epsilon_2 \cdot p_3 \epsilon_1^\mu p_1^\nu, \\ T_9^{\mu\nu} &= \epsilon_2 \cdot p_3 \epsilon_1^\mu p_2^\nu, & T_{10}^{\mu\nu} &= \epsilon_2 \cdot p_3 \epsilon_1^\nu p_1^\mu, & T_{11}^{\mu\nu} &= \epsilon_2 \cdot p_3 \epsilon_1^\nu p_2^\mu, & T_{12}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2^\mu p_1^\nu, \\ T_{13}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2^\mu p_2^\nu, & T_{14}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2^\nu p_1^\mu, & T_{15}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2^\nu p_2^\mu, & T_{16}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2 \cdot p_3 g^{\mu\nu}, \\ T_{17}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2 \cdot p_3 p_1^\mu p_1^\nu, & T_{18}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2 \cdot p_3 p_1^\mu p_2^\nu, \\ T_{19}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2 \cdot p_3 p_2^\mu p_1^\nu, & T_{20}^{\mu\nu} &= \epsilon_1 \cdot p_3 \epsilon_2 \cdot p_3 p_2^\mu p_2^\nu. \end{aligned}$$

HELICITY AMPLITUDES FOR $q\bar{q}' \rightarrow V_1 V_2 \rightarrow l_5 \bar{l}_6 l_7 \bar{l}_8$

$$\mathcal{M}_{\lambda LL}^{V_1 V_2}(p_1, p_2; p_5, p_6, p_7, p_8) = i(4\pi\alpha)^2 \sum_j \frac{L_{l_5 l_6}^{V_1} L_{l_7 l_8}^{V_2} Q_{q q'}^{\lambda, V_1 V_2, [j]}}{D_{V_1}(p_3) D_{V_2}(p_4)} M_{\lambda LL}^{[j]}(p_1, p_2; p_5, p_6, p_7, p_8)$$

where M_{LLL} and M_{RLL} independent, others given by crossing relations. E.g.:

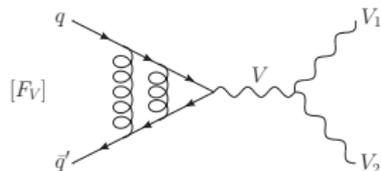
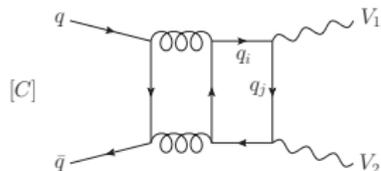
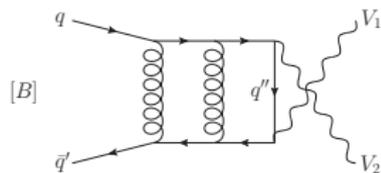
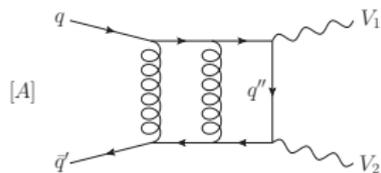
$$\begin{aligned} M_{LLL}(p_1, p_2; p_5, p_6, p_7, p_8) = & [1 \not{p}_3 2] \left\{ E_1 \langle 15 \rangle \langle 17 \rangle [16] [18] \right. \\ & + E_2 \langle 15 \rangle \langle 27 \rangle [16] [28] + E_3 \langle 25 \rangle \langle 17 \rangle [26] [18] \\ & + E_4 \langle 25 \rangle \langle 27 \rangle [26] [28] + E_5 \langle 57 \rangle [68] \left. \right\} \\ & + E_6 \langle 15 \rangle \langle 27 \rangle [16] [18] + E_7 \langle 25 \rangle \langle 27 \rangle [26] [18] \\ & + E_8 \langle 25 \rangle \langle 17 \rangle [16] [18] + E_9 \langle 25 \rangle \langle 27 \rangle [16] [28], \end{aligned}$$

Only 9 out of 10 independent form factors relevant for $d = 4$:

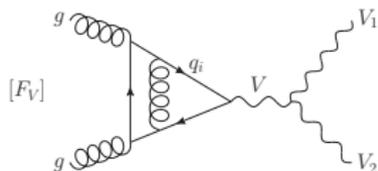
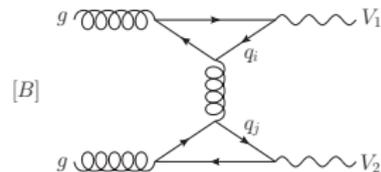
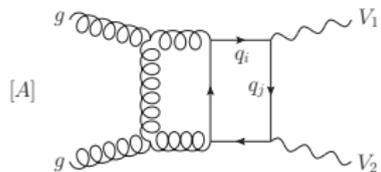
$$\begin{aligned} E_1 &= A_1, & E_6 &= 2A_7 + \frac{2(u - p_3^2)}{s} (A_9 - A_{10}), \\ E_2 &= A_2 + \frac{2}{s} (A_9 - A_{10}), & E_7 &= 2A_8 - \frac{2(t - p_3^2)}{s} (A_9 - A_{10}), \\ E_3 &= A_3 - \frac{2}{s} (A_9 - A_{10}), & E_8 &= 2A_5 - \frac{2}{s} [(u - s - p_3^2)A_9 + (t - p_4^2)A_{10}], \\ E_4 &= A_4, & E_9 &= 2A_6 - \frac{2}{s} [(t - s - p_3^2)A_{10} + (u - p_4^2)A_9]. \\ E_5 &= 2(A_9 + A_{10}), \end{aligned}$$

Feynman Diagrams (generated with Qgraf [Nogueira])

$q\bar{q}'$ channel (just non-zero classes shown):



gg channel ([B] and [FV] do not contribute):

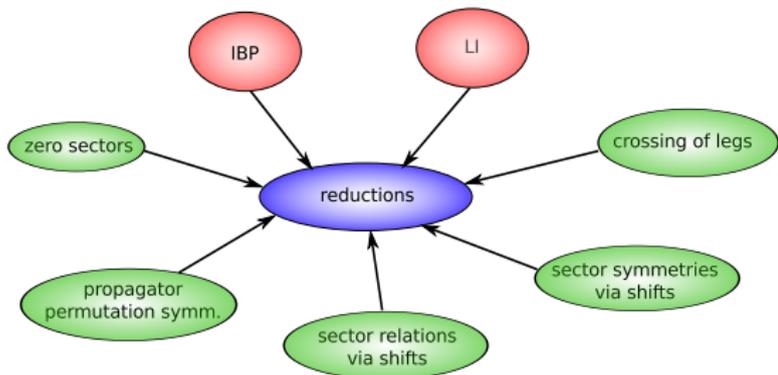




Reduze 2 [AvM, C. Studerus]

arXiv:1201.4330, HepForge

uses GiNaC [Bauer, Frink, Kreckel]
and Fermat [Lewis]

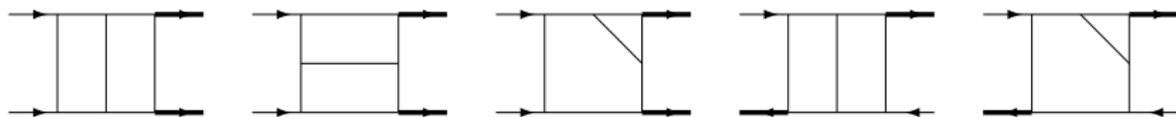


- distributed **Feynman integral reduction**
- advanced **shift finders**
- upcoming version features:
 - ▶ **bilinear propagators**
(3-loop heavy flavour Wilson coefficients in DIS [Blümlein et al. '13-'14])
 - ▶ **phase space integrals**
(soft-virtual N³LO Higgs and DY [Li, AvM, Schabinger, Zhu '14])
 - ▶ **finite integral finder + dimension shifts**
(dims & dots method [AvM, Panzer, Schabinger '14])
 - ▶ **family finder**, ...

MASTER INTEGRALS FOR $q\bar{q}' \rightarrow VV'$ AND $gg \rightarrow VV'$

84 master integrals (w/ products, w/o crossings)

planar two-loop master integrals



non-planar master integrals



METHOD OF DIFFERENTIAL EQUATIONS

- method by [Kotikov '91]; [Gehrmann, Remiddi '99], relies on IBP reduction
- system of differential equations for basis integrals wrt external invariants

$$\frac{\partial}{\partial s_i} I_j(\epsilon, s_m) = \bar{A}_{jk}^{(i)}(\epsilon, s_m) I_k(\epsilon, s_m)$$

- in certain cases proper choice of basis achieves [Kotikov '10]; [Henn '13]:

$$\bar{A}_{jk}^{(i)}(\epsilon, s_m) = \epsilon A_{jk}^{(i)}(s_m)$$

such that

$$dI(\epsilon, s_m) = \epsilon A^{(n)} d \ln I_n(s_m) I(\epsilon, s_m)$$

with full decoupling after expansion in $\epsilon = (4 - d)/2$

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obtain **pure functions** due to particular choice of basis:

- pure functions \Leftrightarrow every term of ϵ expansion has uniform weight
- applies to phase space integrals [Hörschle, Hoff, Ueda '14]; [AvM, Schabinger, Zhu '14]
- useful for exact d dependent single scale integrals [Li, AvM, Schabinger, Zhu '14]
- construction of canonical form: [Henn, Smirnov, Smirnov '13]; [Gehrmann, AvM, Tancredi, Weihs '14]; [Lee '14]

STRUCTURE OF RESULT

vector of 111 master integrals in canonical basis with alphabet:

$$\{\bar{l}_1, \dots, \bar{l}_{20}\} = \{2, \bar{x}, 1 + \bar{x}, 1 - \bar{y}, \bar{y}, 1 + \bar{y}, 1 - \bar{x}\bar{y}, 1 + \bar{x}\bar{y}, 1 - \bar{z}, \bar{z}, \\ 1 + \bar{y} - 2\bar{y}\bar{z}, 1 - \bar{y} + 2\bar{y}\bar{z}, 1 + \bar{x}\bar{y} - 2\bar{x}\bar{y}\bar{z}, 1 - \bar{x}\bar{y} + 2\bar{x}\bar{y}\bar{z}, \\ 1 + \bar{y} + \bar{x}\bar{y} + \bar{x}\bar{y}^2 - 2\bar{y}\bar{z} - 2\bar{x}\bar{y}\bar{z}, 1 + \bar{y} - \bar{x}\bar{y} - \bar{x}\bar{y}^2 - 2\bar{y}\bar{z} + 2\bar{x}\bar{y}\bar{z}, \\ 1 - \bar{y} - \bar{x}\bar{y} + \bar{x}\bar{y}^2 + 2\bar{y}\bar{z} + 2\bar{x}\bar{y}\bar{z}, 1 - \bar{y} + \bar{x}\bar{y} - \bar{x}\bar{y}^2 + 2\bar{y}\bar{z} - 2\bar{x}\bar{y}\bar{z}, \\ 1 - 2\bar{y} - \bar{x}\bar{y} + \bar{y}^2 + 2\bar{x}\bar{y}^2 - \bar{x}\bar{y}^3 + 4\bar{y}\bar{z} + 2\bar{x}\bar{y}\bar{z} + 2\bar{x}\bar{y}^3\bar{z}, \\ 1 - \bar{y} - 2\bar{x}\bar{y} + 2\bar{x}\bar{y}^2 + \bar{x}^2\bar{y}^2 - \bar{x}^2\bar{y}^3 + 2\bar{y}\bar{z} + 4\bar{x}\bar{y}\bar{z} + 2\bar{x}^2\bar{y}^3\bar{z}\}$$

in parametrisation which rationalizes root of Källén function $\sqrt{s^2 + p_3^4 + p_4^4 - 2(s p_3^2 + p_3^2 p_4^2 + p_4^2 s)}$:

$$s = \bar{m}^2(1 + \bar{x})^2, \quad t = -\bar{m}^2\bar{x}((1 + \bar{y})(1 + \bar{x}\bar{y}) - 2\bar{z}\bar{y}(1 + \bar{x})), \quad p_3^2 = \bar{m}^2\bar{x}^2(1 - \bar{y}^2), \quad p_4^2 = \bar{m}^2(1 - \bar{x}^2\bar{y}^2)$$

integrated in terms of:

MULTIPLE POLYLOGARITHMS [REMIDI, GEHRMANN]; [GONCHAROV]

$$G(a_1, a_2, \dots, a_n; x) = \int_0^x dt \frac{dt}{t - a_1} G(a_2, \dots, a_n; t),$$

- independent input for a couple of very simple bubbles and triangles
- remaining boundary functions fixed by regularity
- coproduct and more [Brown '11], [Duhr '12], [Duhr, Gangl, Rhodes '11], [Vollinga, Weinzierl '04]

choose real valued $\ln l_i$, $\text{Li}_n(R_1)$, $\text{Li}_{2,2}(R_1, R_2)$ with

$$|R_1| < 1, \quad |R_1 R_2| < 1$$

where R_i are power products of letters (e.g. $-l_1, l_3, -l_8/(l_1 l_3), \dots$)

such that Li functions have convergent power series

$$\text{Li}_n(R_1) = - \sum_{j_1=1}^{\infty} \frac{R_1^{j_1}}{j_1^n}, \quad \text{Li}_{2,2}(R_1, R_2) = \sum_{j_1=1}^{\infty} \sum_{j_2=1}^{\infty} \frac{R_1^{j_1}}{(j_1 + j_2)^2} \frac{(R_1 R_2)^{j_2}}{j_2^2}$$

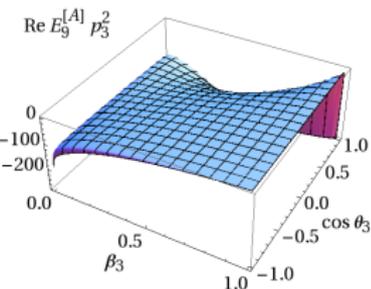
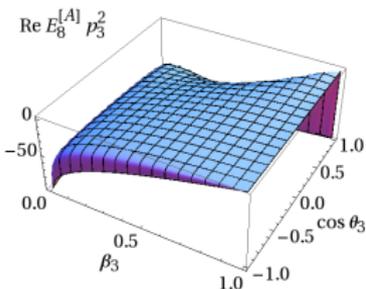
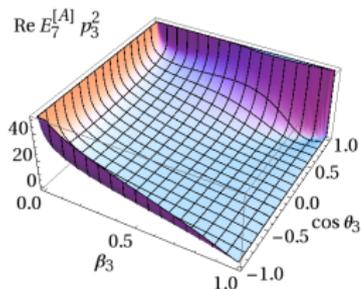
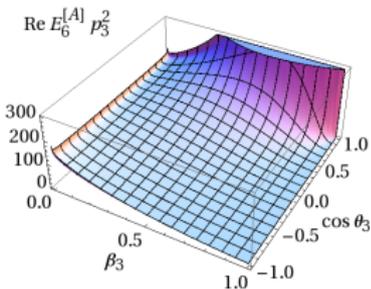
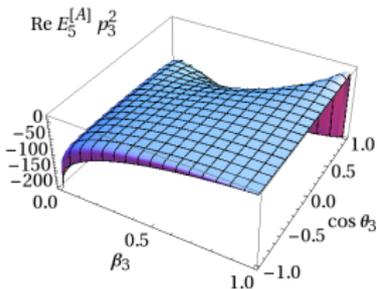
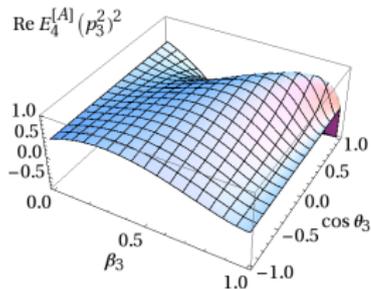
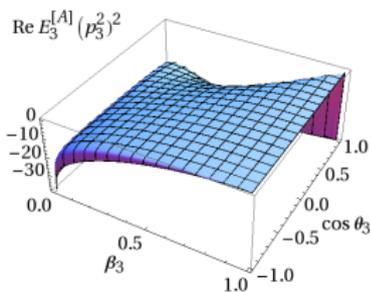
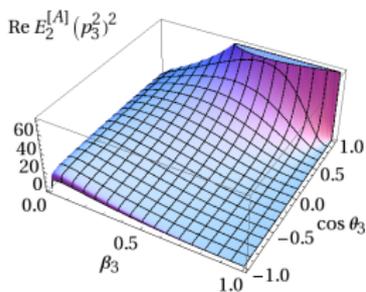
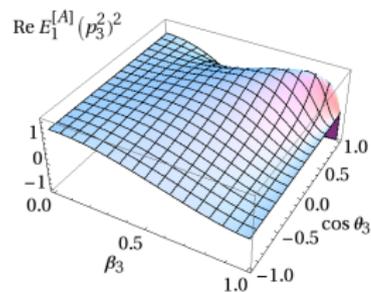
features:

- argument construction based on [Duhr, Gangl, Rhodes '11]
- no spurious letters, no artificial linearisation
- parametrisation $s = m^2(1+x)(1+xy)$, $t = -m^2xz$, $p_3^2 = m^2$, $p_4^2 = m^2x^2y$: shorter alphabet

$$\begin{aligned} \{l_1, \dots, l_{17}\} = \{ & x, 1+x, y, 1-y, z, 1-z, -y+z, 1+y-z, 1+xy, 1+xz, xy+z, \\ & 1+y+xy-z, 1+x+xy-xz, 1+y+2xy-z+x^2yz, \\ & 2xy+x^2y+x^2y^2+z-x^2yz, 1+x+y+xy+xy^2-z-xz-xyz, \\ & 1+y+xy+y^2+xy^2-z-yz-xyz \} \end{aligned}$$

- **very fast and stable** numerical evaluation: $O(150ms)$ full amplitude ($O(35ms)$ for $p_3^2 = p_4^2$)
- remark: can actually skip “traditional integration”

helicity amplitudes for $q\bar{q}' \rightarrow VV'$ @ 2-loops [Gehrmann, AvM, Tancredi '15]

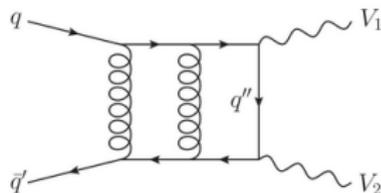


VVamp project

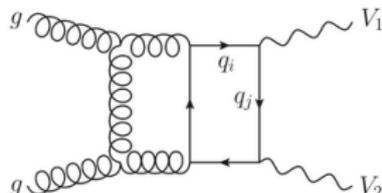
This is the web page of the VVamp project. We provide the two-loop helicity amplitudes for electroweak vector boson pair production and their decay into 4 leptons in quark-antiquark annihilation and in gluon-gluon fusion.

You can download our analytical results for the master integrals and the amplitudes. Moreover, we provide C++ implementations for the fast and reliable numerical evaluation of the amplitudes.

Quark channel



Gluon channel



Reference

- Thomas Gehrmann, Andreas von Manteuffel, Lorenzo Tancredi: "The two-loop helicity amplitudes for $q\bar{q}' \rightarrow V1V2 \rightarrow 4$ leptons", [arXiv:1503.04812](https://arxiv.org/abs/1503.04812)

Downloads: amplitudes

- bare form factors exact in d: `class A`, `class B`, `class C` (Form format)
- finite form factors in qt-scheme: `class A`, `class B`, `class C` (Form format)
- relations for projectors: A_j of τ and τ of A_j (Form format)
- numerical implementation of form factors: `qqvvamp` package (C++, requires `GiNaC`)

Downloads: master integrals

- master integral definitions: `Mathematica`, Form format
- master integral traditional solutions: `Mathematica`, Form format
- master integral optimised solutions: `Mathematica`, Form format
- master integral crossing relations: `Mathematica`, Form format
- integral families, kinematics (in `Reduze 2` format)

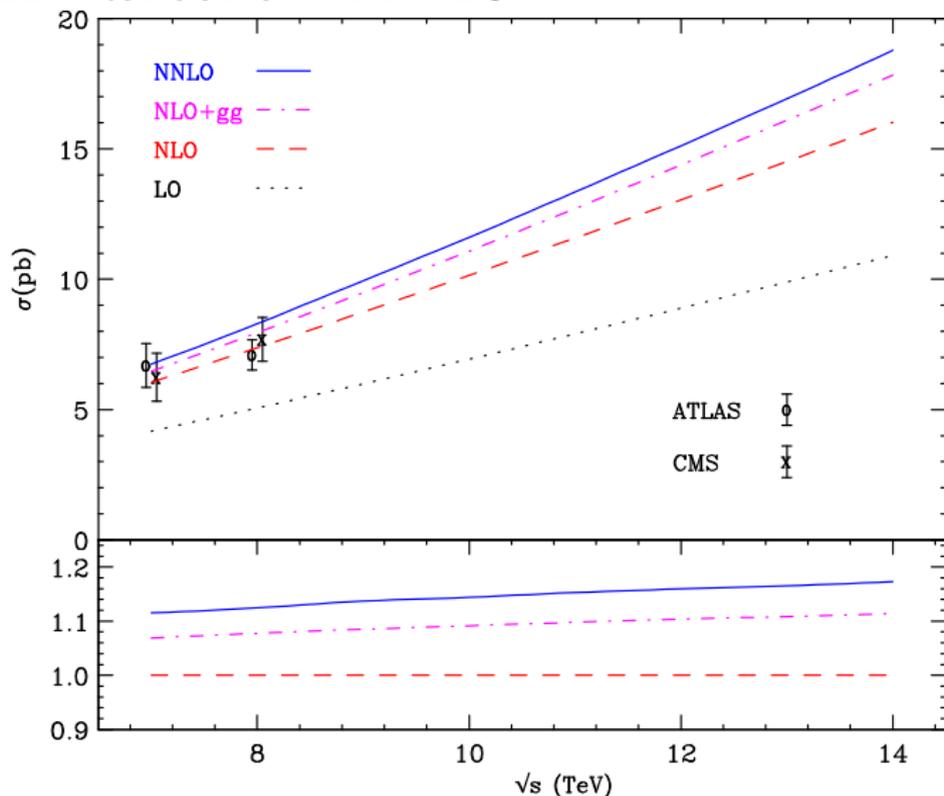
Reference

- Andreas von Manteuffel, Lorenzo Tancredi: "The two-loop helicity amplitudes for $g\bar{g} \rightarrow V1V2 \rightarrow 4$ leptons", [arXiv:1503.08835](https://arxiv.org/abs/1503.08835)

Downloads: amplitudes

- bare form factors exact in d: `class A` (Form format)
- finite form factors in qt-scheme: `class A` (Form format)
- relations for projectors: P_j of T_j and T_j of P_j (Form format)
- numerical implementation of form factors: `ggvvamp` package (C++, requires `GiNaC`)

RESULT: ZZ PRODUCTION AT NNLO



- NNLO corrections: 11%-17%
- gg contributes 60% of NNLO

[Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi, Weihs '14]

ZZ PRODUCTION: SCALE UNCERTAINTIES

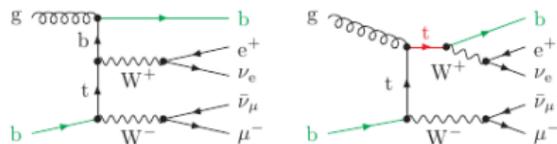
\sqrt{s} (TeV)	σ_{LO} (pb)	σ_{NLO} (pb)	σ_{NNLO} (pb)
7	4.167 ^{+0.7%} _{-1.6%}	6.044 ^{+2.8%} _{-2.2%}	6.735 ^{+2.9%} _{-2.3%}
8	5.060 ^{+1.6%} _{-2.7%}	7.369 ^{+2.8%} _{-2.3%}	8.284 ^{+3.0%} _{-2.3%}
9	5.981 ^{+2.4%} _{-3.5%}	8.735 ^{+2.9%} _{-2.3%}	9.931 ^{+3.1%} _{-2.4%}
10	6.927 ^{+3.1%} _{-4.3%}	10.14 ^{+2.9%} _{-2.3%}	11.60 ^{+3.2%} _{-2.4%}
11	7.895 ^{+3.8%} _{-5.0%}	11.57 ^{+3.0%} _{-2.4%}	13.34 ^{+3.2%} _{-2.4%}
12	8.882 ^{+4.3%} _{-5.6%}	13.03 ^{+3.0%} _{-2.4%}	15.10 ^{+3.2%} _{-2.4%}
13	9.887 ^{+4.9%} _{-6.1%}	14.51 ^{+3.0%} _{-2.4%}	16.91 ^{+3.2%} _{-2.4%}
14	10.91 ^{+5.4%} _{-6.7%}	16.01 ^{+3.0%} _{-2.4%}	18.77 ^{+3.2%} _{-2.4%}

- variation: $0.5m_Z < \mu_R, \mu_F < 2m_Z$ with $0.5 < \mu_F/\mu_R < 2$
- scale uncertainty 3% not decreased

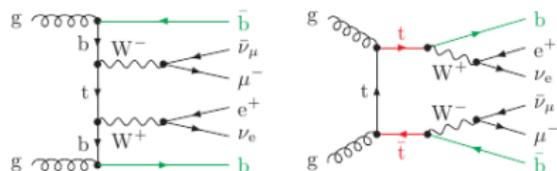
Definition of top-contamination free WW cross section in 5FNS

Definition of WW cross section beyond LO

- straightforward in 4FNS (massive b's)
- non-trivial in 5FNS (massless b's)
 - Single-top production enters at NLO.



- Top-pair production enters at NNLO.

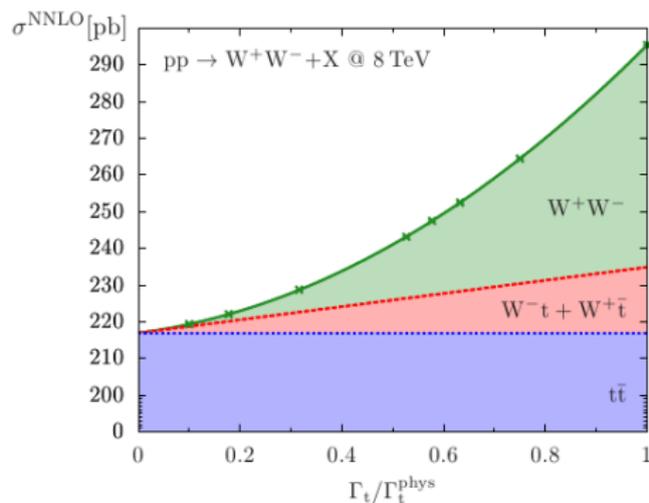


⇒ Huge "higher-order corrections" from top-resonance contamination in 5FNS.

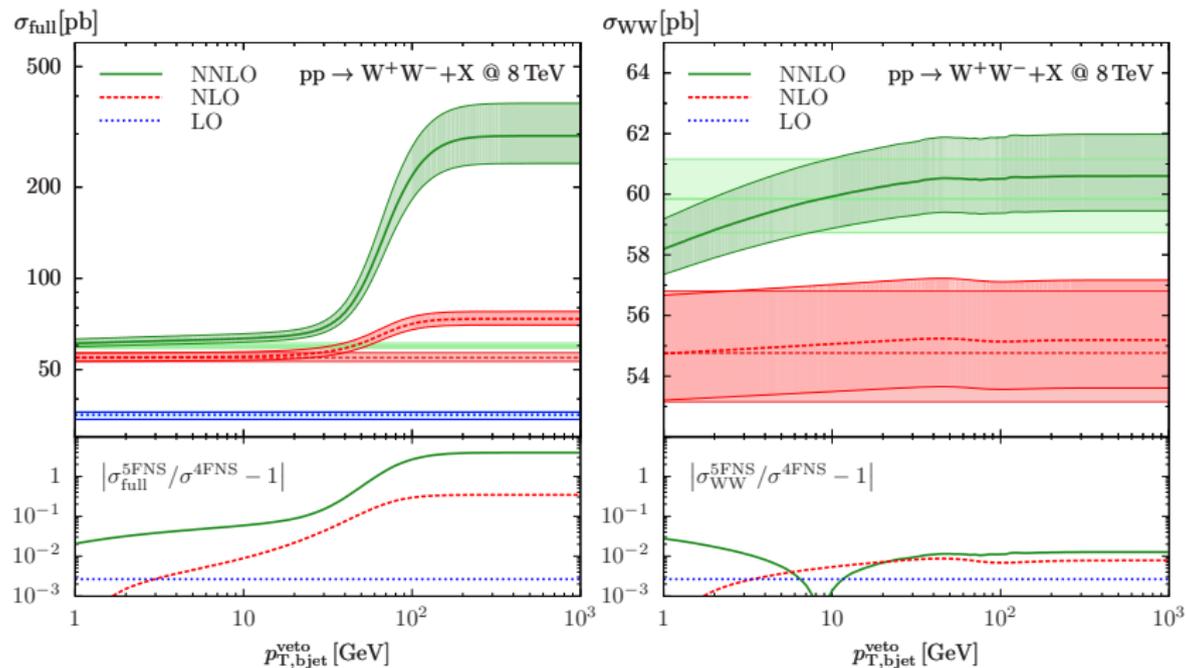
Γ_t -dependence of NNLO cross section can be used to isolate the different processes:

$$\sigma_{WW} \propto 1, \quad \sigma_{tW} \propto 1/\Gamma_t, \quad \sigma_{t\bar{t}} \propto 1/\Gamma_t^2.$$

⇒ Parabolic fit of the $(\Gamma_t/\Gamma_t^{\text{phys}})^2$ -rescaled cross section delivers σ_{WW} , σ_{tW} , $\sigma_{t\bar{t}}$.



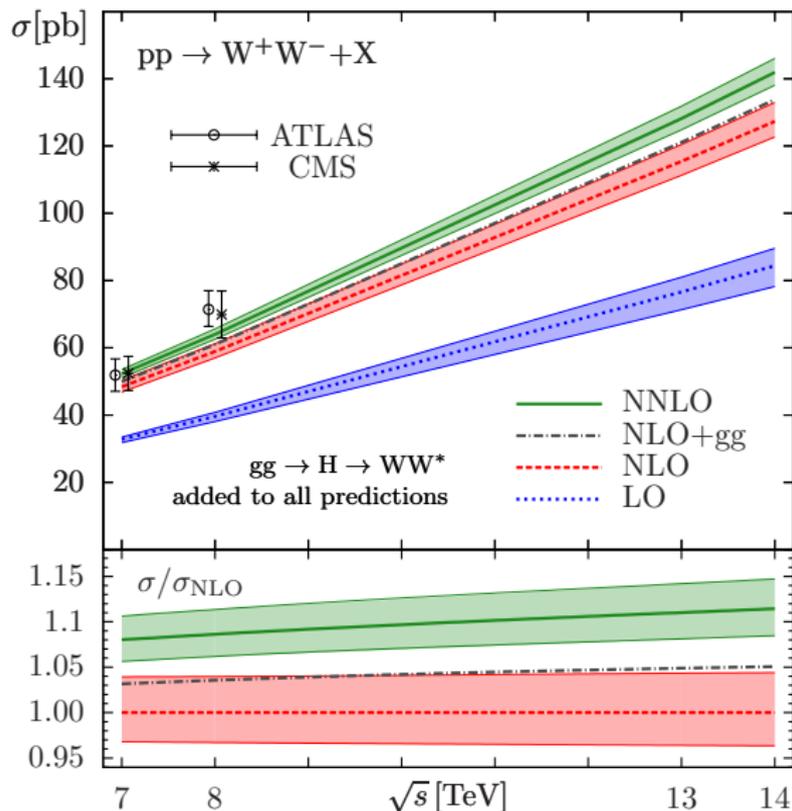
NEW DEFINITION OF W^+W^- CROSS SECTION



- top-subtracted WW cross section: robust & precise

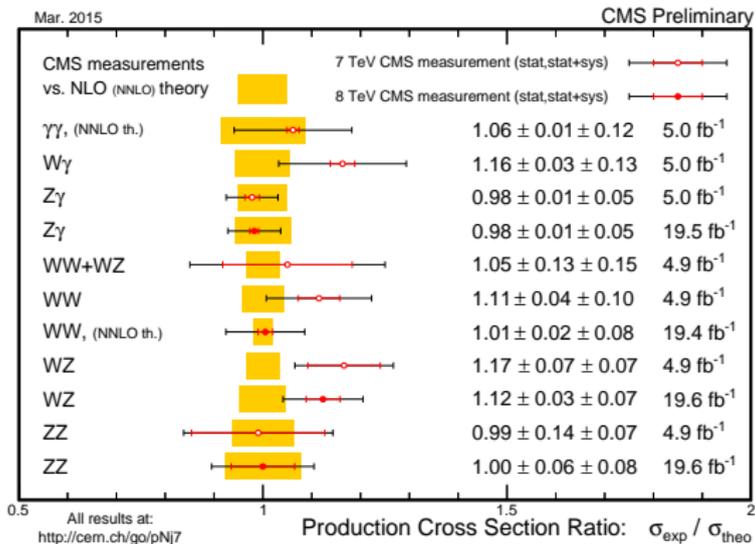
[Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14]

RESULT: W^+W^- PRODUCTION AT NNLO



[Gehrmann, Grazzini, Kallweit, Maierhöfer, AvM, Pozzorini, Rathlev, Tancredi '14]

RECENT UPDATE BY CMS



CONCLUSIONS & OUTLOOK

- $pp \rightarrow VV' \rightarrow 4 \text{ leptons}$ @ NNLO

- ▶ complete set of two-loop master integrals
- ▶ improved treatment of multiple polylogarithms
- ▶ analytic two-loop helicity amplitudes: fast & precise
- ▶ NNLO prediction for ZZ and W^+W^- production at LHC
- ▶ new definition of W^+W^- cross section: top-subtraction

- outlook

- ▶ fiducial cross sections (cmp. [Monni, Zanderighi '14])
- ▶ differential distributions, off-shell effects
- ▶ NNLL+NNLO p_T resummation
- ▶ WZ production
- ▶ $gg \rightarrow VV'$ @ NLO
- ▶ new results coming soon: see talks by [Rathlev], [Wiesemann] at RadCor-LoopFest '15

SUPPLEMENTARY SLIDES

- 2 WW PRODUCTION: SCALE UNCERTAINTIES
- 3 CANONICAL BASIS CONSTRUCTION
- 4 ALGORITHMS FOR MULTIPLE POLYLOGARITHMS
- 5 COPRODUCT FOR MULTIPLE POLYLOGARITHMS
- 6 EXAMPLE FOR SYMBOL CALCULUS

W^+W^- PRODUCTION AT NNLO: SCALE UNCERTAINTIES

$\frac{\sqrt{s}}{\text{TeV}}$	σ_{LO}	σ_{NLO}	σ_{NNLO}	$\sigma_{gg \rightarrow H \rightarrow WW^*}$
7	$29.52^{+1.6\%}_{-2.5\%}$	$45.16^{+3.7\%}_{-2.9\%}$	$49.04^{+2.1\%}_{-1.8\%}$	$3.25^{+7.1\%}_{-7.8\%}$
8	$35.50^{+2.4\%}_{-3.5\%}$	$54.77^{+3.7\%}_{-2.9\%}$	$59.84^{+2.2\%}_{-1.9\%}$	$4.14^{+7.2\%}_{-7.8\%}$
13	$67.16^{+5.5\%}_{-6.7\%}$	$106.0^{+4.1\%}_{-3.2\%}$	$118.7^{+2.5\%}_{-2.2\%}$	$9.44^{+7.4\%}_{-7.9\%}$
14	$73.74^{+5.9\%}_{-7.2\%}$	$116.7^{+4.1\%}_{-3.3\%}$	$131.3^{+2.6\%}_{-2.2\%}$	$10.64^{+7.5\%}_{-8.0\%}$

ROTATING TO CANONICAL BASIS

how to find **canonical basis** ?

- some heuristics in [Smirnov, Smirnov, Henn '13]: cuts, explicite bubble insertions

our proposal:

PROCEDURE

construct canonical basis starting from rough first guess

- 1 **bottom up** strategy, assume subtopos in canonical form
- 2 for given sector, guess basis: **triangular for $\epsilon = 0$** , diff. eq. **linear in ϵ** (top level only)
- 3 integrate out **homogeneous part for $\epsilon = 0$** (top level only)
- 4 remove unwanted terms $1/(u - v\epsilon)^n$, 1 , ϵ^n iteratively
simplifying assumption: restrict to minimal shifts

result: canonical basis

very recently: new method by [Lee '14]

[[Remiddi, Gehrmann; Goncharov]]

$$G(a_1, a_2, \dots, a_n; x) = \int_0^x dt \frac{dt}{t - a_1} G(a_2, \dots, a_n; t),$$

with $G(; x) = 1$, complex weights a_i and complex argument x .

we employ also **generalised weights** $[f(o)]$:

$$G([f(o)], w_2, \dots, w_n; x) = \int_0^x dt \frac{f'(t)}{f(t)} G(w_2, \dots, w_n; t)$$

example:

$$G([o^2 + 1]; x) = \int_0^x dt \frac{2t}{t^2 + 1} = \int_0^x dt \frac{1}{t - i} + \int_0^x dt \frac{1}{t + i} = G(i; x) + G(-i; x)$$

see [AvM, Schabinger, Zhu '13], related: [Ablinger, Blümlein, Schneider '11] (cyclotomic polylogs)

we apply coproduct based and other algorithms

- partly from [Brown '11], [Duhr '12], [Duhr, Gangl, Rhodes '11]
- numerical routines from [Vollinga, Weinzierl '04]

main algorithms:

1 normal form for specific arguments

- ▶ independent of symbol calculus
- ▶ uses [Vollinga, Weinzierl '04] for numerical evaluation, fits constants

2 coproduct based normal form for general choice of basis

- ▶ based on [Goncharov '02], [Brown '11], [Duhr '12], [Duhr, Gangl, Rhodes '11]
- ▶ handles generalised weights
- ▶ identifies products (e.g. $G(0, 1; x) + G(1, 0; x) \rightarrow G(0; x)G(1; x)$)
- ▶ matches irreducible factors *at symbol level*
- ▶ uses [Vollinga, Weinzierl '04] for numerical evaluation, fits constants

3 construct new basis with desired properties

- ▶ based on [Duhr, Gangl, Rhodes '11], apply to generalised weights

DEFINITION OF THE COPRODUCT

For a multiple polylogarithm

$$I(a_0; a_1, \dots, a_n; a_{n+1}) = \int_{a_0}^{a_{n+1}} \frac{dt}{t - a_n} I(a_0; a_1, \dots, a_{n-1}; t)$$

the coproduct Δ is defined according to [Goncharov ('02)]:

$$\Delta(I(a_0; a_1, \dots, a_n; a_{n+1})) = \sum_{0=i_1 < \dots < i_{k+1}=n} I(a_0; a_{i_1}, \dots, a_{i_k}; a_{n+1}) \otimes \prod_{p=0}^k I(a_{i_p}; a_{i_{p+1}}, \dots, a_{i_{p+1}-1}; a_{i_{p+1}})$$

examples:

- $\Delta(\ln(x)) = 1 \otimes \ln(x) + \ln(x) \otimes 1$
- $\Delta(\text{Li}_2(x)) = 1 \otimes \text{Li}_2(x) - \ln(1-x) \otimes \ln(x) + \text{Li}_2(x) \otimes 1$
- $\Delta(\ln(x) \ln(y)) = 1 \otimes (\ln(x) \ln(y)) + \ln(x) \otimes \ln(y) + \ln(y) \otimes \ln(x) + (\ln(x) \ln(y)) \otimes 1$

RULES FOR THE COPRODUCT

- coassociativity $(\text{id} \otimes \Delta) \Delta = (\Delta \otimes \text{id}) \Delta$
- compatible with product: $\Delta(a \cdot b) = \Delta(a) \cdot \Delta(b)$ where $(a_1 \otimes a_2) \cdot (b_1 \otimes b_2) \equiv (a_1 \cdot b_1) \otimes (a_2 \cdot b_2)$

note: coproduct means "decomposition"

GRADED DECOMPOSITION WITH THE COPRODUCT

- Hopf algebra of multiple polylogs **graded by weight**:

$$\mathcal{H} = \bigoplus_{n=0}^{\infty} \mathcal{H}_n$$

since coproduct preserves weight we may decompose

$$\mathcal{H}_n \xrightarrow{\Delta} \bigoplus_{p+q=n} \mathcal{H}_p \otimes \mathcal{H}_q$$

and define $\Delta_{p,q}$ to be the part with values in $\mathcal{H}_p \otimes \mathcal{H}_q$

- iterated coproduct:

$$\mathcal{H} \xrightarrow{\Delta} \mathcal{H} \otimes \mathcal{H} \xrightarrow{\Delta \otimes \text{id}} \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} \xrightarrow{\Delta \otimes \text{id} \otimes \text{id}} \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$$

and corresponding parts $\Delta_{p,q,\dots,r}$

- symbol $\mathcal{S} = \text{maximally iterated coproduct } \Delta_{1,\dots,1} \pmod{\pi}$

EXAMPLE FOR SYMBOL CALCULUS

goal: derive "simplification formula" for $\text{Li}_2(1/x)$ with $0 < x < 1$, $\text{Im } x = \varepsilon$

$$\begin{aligned}\mathcal{S}(\text{Li}_2(1/x)) &= -(-1 + 1/x) \otimes (1/x) \\ &= (1 - x) \otimes x - x \otimes x \\ &= \mathcal{S}(-\text{Li}_2(x) - (1/2) \ln^2 x)\end{aligned}$$

reproduces the highest degree part of the full answer

$$\text{Li}_2(1/x) = -\text{Li}_2(x) - (1/2) \ln^2 x + i\pi \ln x - (2/3)\pi^2$$

note: works at **highest degree** only

- $\mathcal{S}(\ln(-x)) = \mathcal{S}(\ln(x))$: no info on discontinuity
- $\mathcal{S}(\pi) = \mathcal{S}(\zeta_3) = 0$: no constants

"integrating the symbol" \Rightarrow **algorithmic reduction** (structured with shuffle eliminators)
[Duhr, Gangl, Rhodes ('11)]

What about **subleading degree** terms ($\text{const} \times \text{polylog}$) ?

- accessible by **coproduct**: [Goncharov ('02), Brown ('11)]
- coproduct means “decomposition”
- symbol \mathcal{S} = maximally iterated coproduct $\Delta_{1,\dots,1} \bmod \pi$
- extended symbol calculus based on coproduct by [Duhr ('12)] with:

$$\Delta(\pi) = \pi \otimes 1$$

$$\Delta(\zeta_k) = \zeta_k \otimes 1 + 1 \otimes \zeta_k \quad \text{for } k \text{ (odd)}$$

example:

$$\begin{aligned} \Delta_{1,1}(\text{Li}_2(1/x)) &= -\ln(1 - 1/x) \otimes \ln(1/x) \\ &= \ln(1 - x) \otimes \ln(x) - \ln(x) \otimes \ln(x) + i\pi \otimes \ln(x) \\ &= \Delta_{1,1}(-\text{Li}_2(x) - (1/2)\ln^2(x) + i\pi \ln(x)) \end{aligned}$$

reproduces identity up to pure constant, fix by limits or numerical evaluation:

$$\text{Li}_2(1/x) - (-\text{Li}_2(x) - (1/2)\ln^2(x) + i\pi \ln(x)) = -6.5797362673929 \dots = -(2/3)\pi^2$$